

Summer Math

I hope you are excited for the year of **AP Calculus** that we will be pursuing together. I don't know how much you know about Calculus but it is not like any other math that you have learned so far. For most of the first semester we will be working on derivatives and the second semester we will be doing integrals. You don't need to know about those yet but I will tell you that Calculus is described as the mathematics of change – how fast things change, how to predict change and how to use information about change to understand the functions themselves.

In some ways, Calculus is taking what you already know one step further. Previous courses taught you how to find the slope of a line. Calculus teaches you how to find the slope of a curve. Previous courses have taught you how to find the length of a straight rope. Calculus will teach you how to find the length of a curved rope. Previous courses have taught you how to find the area of a rectangular roof. Calculus will teach you how to find the area of a curved dome-shaped roof.

You may already be wondering how we will figure this out. Imagine a curve like this:



If you were to zoom in a few times, each part of the curve would kind of look like a line. If a few times wasn't enough, then you could zoom in more and more and more. The process of zooming in an infinite number of times is the foundation of Calculus. This process is called a "limit" and that is where we will begin.

In preparations for **AP Calculus**, I have prepared a review of concepts for summer review. These are concepts that you have been taught in previous math classes and problems that you should know how to do. This packet does not require you to use a calculator; in fact *you should not use a calculator at all on any of these problems*. AP Calculus builds on the concepts in this packet. I expect you to know the concepts in this packet in order to help you be successful in AP Calculus. The first two pages are general material that could help you answer questions on the remainder of the pages. Most pages begin with some examples to help refresh your memory on that topic.

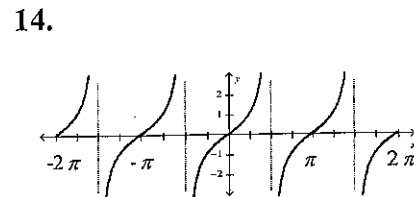
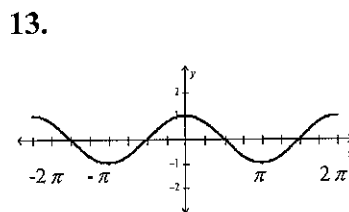
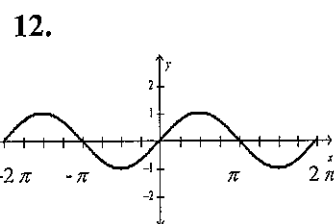
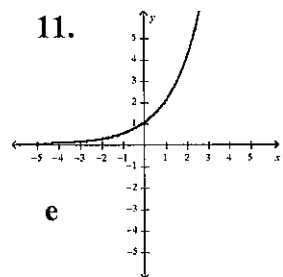
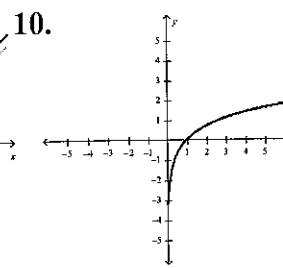
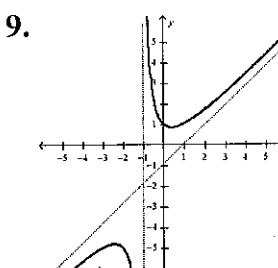
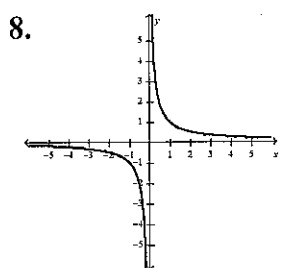
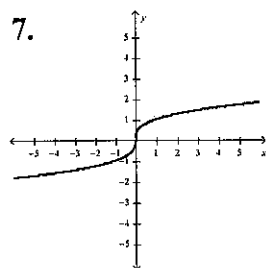
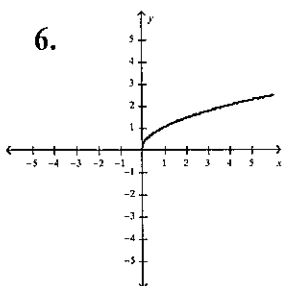
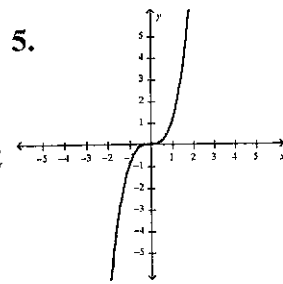
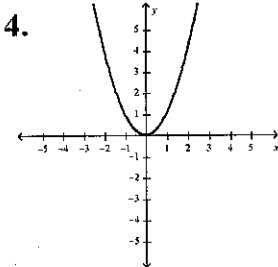
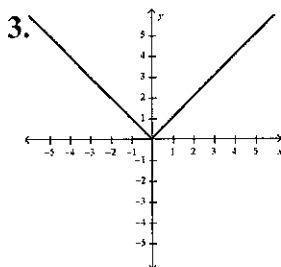
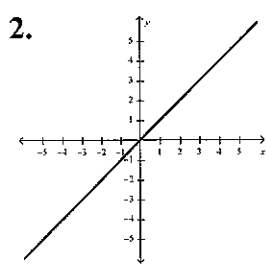
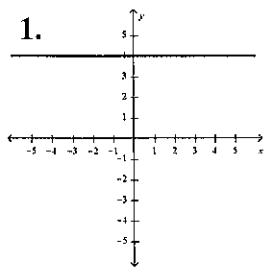
If you are struggling with this work you can get help from a friend, parent, or tutor. Additionally, you may find websites that can be helpful if you search for the information on the topics listed at the top of the page. Several pages begin with an example to refresh your memory. While you may get help, you *are expected to do your own work!!* Please show all work on this packet or on a separate piece of paper to be attached when you turn it in on the first day of school. Make sure your work is organized and neatly written.

Additionally, you will have a graded assignment within the first two weeks of school. The graded assignment will cover all the concepts in this packet, but will not be the exact same problems. The graded assignment will also include some material from the first unit that we will cover. This assignment will be completed without a calculator.

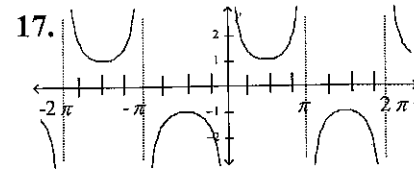
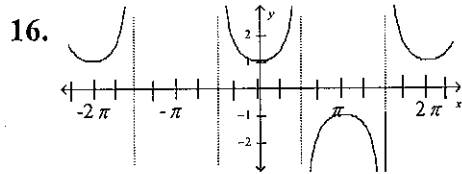
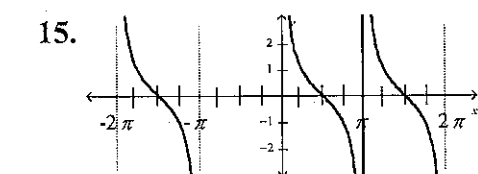
Student Name

PARENT FUNCTIONS: Write the letter of the function and the number of the graph in the blank under the name of the appropriate model.

- a. $f(x) = \cot x$ b. $f(x) = x^2$ c. $f(x) = \tan x$ d. $f(x) = \log_a x$ e. $f(x) = x$
 f. $f(x) = \frac{(x^2+1)(x-2)}{(x+1)(x-2)}$ g. $f(x) = a^x$ h. $f(x) = \sqrt[3]{x}$ i. $f(x) = \sqrt{x}$ j. $f(x) = a$
 k. $f(x) = \sin x$ l. $f(x) = \cos x$ m. $f(x) = |x|$ n. $f(x) = x^3$
 o. $f(x) = \frac{1}{x}$ p. $f(x) = \sec x$ q. $f(x) = \csc x$

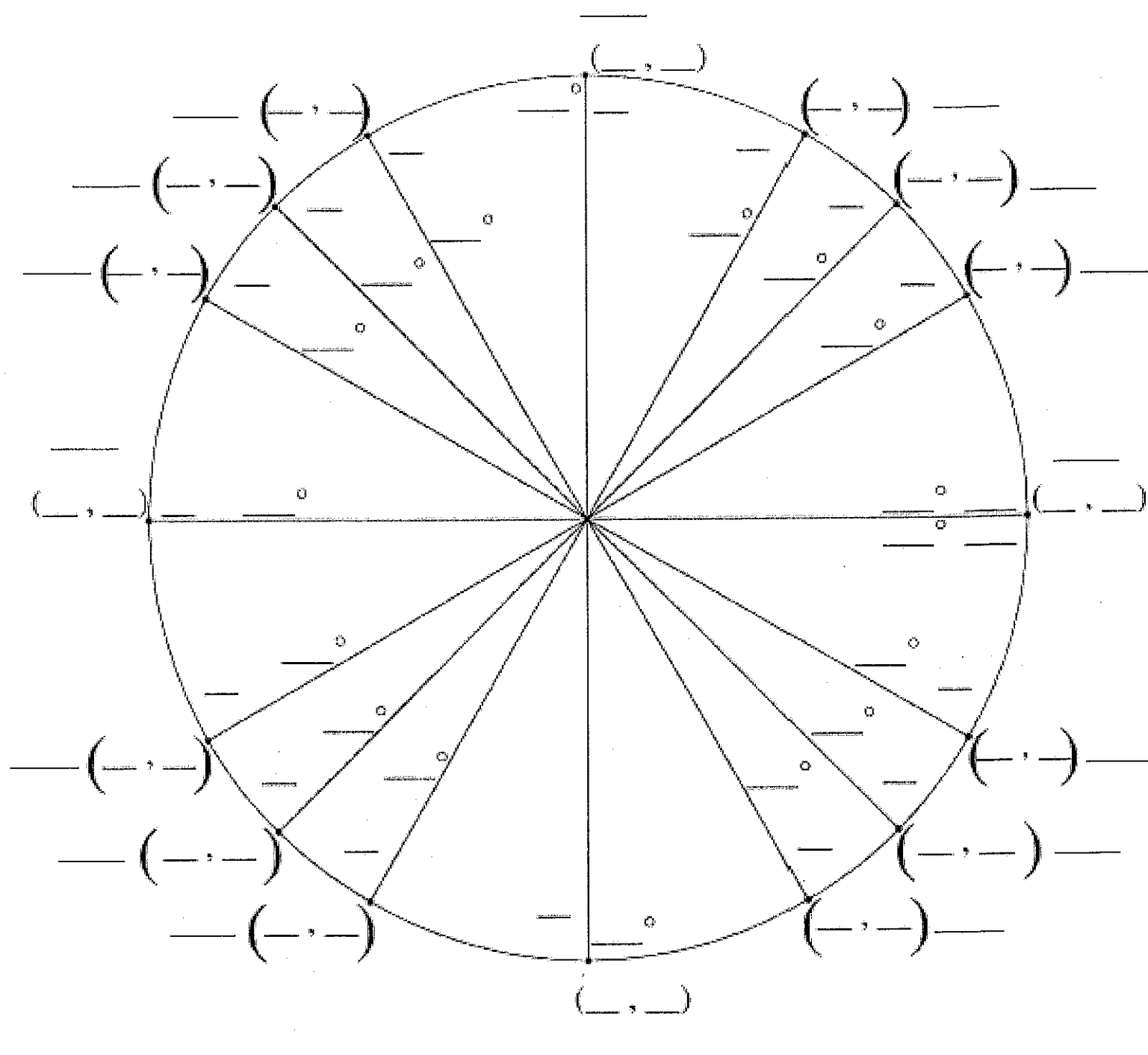


e



Absolute Value	Constant	Cube Root	Exponential
_____	_____	_____	_____
Linear	Quadratic	Rational	Square Root
_____	_____	_____	_____
Reciprocal (Inverse Power)	Trigonometric (six functions)		
_____	_____	_____	_____
_____	_____	_____	_____

Fill in The Unit Circle



EmbeddedMath.com

REMEMBER:

(cos, sin) tan

FUNCTIONS:

To evaluate a function for a given value, simply plug the value of the function in for x .

Remember: $(f \circ g)(x) = f(g(x))$ or $f[g(x)]$

Example: $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ Find $f(g(x))$.

$$\begin{aligned} f(g(x)) &= f(x - 4) = 2(x - 4)^2 + 1 = 2(x^2 - 8x + 16) + 1 = 2x^2 - 16x + 32 + 1 \\ &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Simplify where necessary.

1. $f(2) =$ _____ 2. $g(-3) =$ _____ 3. $f(h + 2) =$ _____

4. $f[g(-2)] =$ _____ 5. $g[f(m + 2)] =$ _____

6. $[f(x)]^2 - 2g(x) =$ _____ 7. $\frac{f(x + h) - f(x)}{h} =$ _____

Find $\frac{f(x + h) - f(x)}{h}$ for the following.

8. $f(x) = 9x + 3$ 9. $f(x) = 5 - 2x$

Let $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = \sin(2x)$. Find each EXACT value.

10. $f(\pi) =$ _____ 11. $g(\pi) =$ _____ 12. $f\left(\frac{\pi}{4}\right) =$ _____ 13. $h\left(\frac{\pi}{4}\right) =$ _____

14. $g\left(\frac{3\pi}{2}\right) =$ _____ 15. $f\left(\frac{2\pi}{3}\right) =$ _____ 16. $h\left(\frac{2\pi}{3}\right) =$ _____

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$.

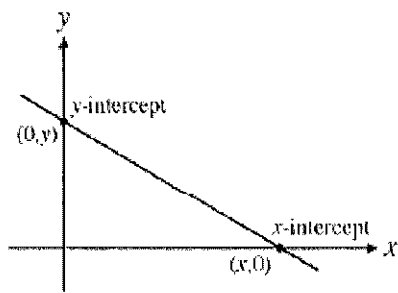
17. $h[f(-2)] =$ _____ 18. $f[g(x - 1)] =$ _____

19. $g[h(x^3)] =$ _____ 20. $f[g[h(2x)]] =$ _____

INTERCEPTS OF A GRAPH:

To find the x-intercepts, let $y = 0$ in your equation and solve.

To find the y-intercepts, let $x = 0$ in your equation and solve.



Example: Given the function $y = x^2 - 2x - 3$, find all intercepts.

x-int. (Let $y = 0$)

$$0 = x^2 - 2x - 3$$

$$0 = (x - 3)(x + 1)$$

$$x = -1 \text{ or } x = 3$$

x-intercepts $(-1, 0)$ and $(3, 0)$

y-int. (Let $x = 0$)

$$y = 0^2 - 2(0) - 3$$

$$y = -3$$

y-intercept $(0, -3)$

Find the x and y intercepts for each.

1. $y = 2x - 5$

2. $y = x^2 + x - 2$

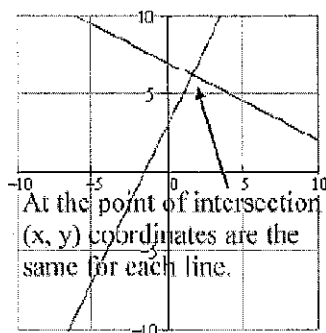
3. $y = x\sqrt{16 - x^2}$

4. $y^2 = x^3 - 4x$

POINTS OF INTERSECTION:

Use substitution or elimination method to solve the system of equations.

Remember: You are finding a POINT OF INTERSECTION so your answer is an ordered pair.



CALCULATOR TIP

Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2nd Trace) and hit INTERSECT.

Example: Find all points of intersection of $x^2 - y = 3$
 $x - y = 1$

ELIMINATION METHOD

Subtract to eliminate y

$$x^2 - x = 2$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \text{ or } x = -1$$

Plug in $x = 2$ and $x = -1$ to find y

Points of Intersection: (2, 1) and (-1, -2)

SUBSTITUTION METHOD

Solve one equation for one variable.

$$y = x^2 - 3$$

$$y = x - 1$$

Therefore by substitution $x^2 - 3 = x - 1$

$$x^2 - x - 2 = 0$$

From here it is the same as the other example

Find the point(s) of intersection for the given equations.

1. $x + y = 8$
 $4x - y = 7$

2. $x^2 + y = 6$
 $x + y = 4$

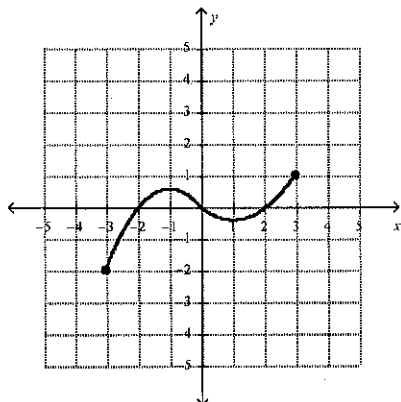
3. $x = 3 - y^2$
 $y = x - 1$

DOMAIN AND RANGE:

Domain – all x values for which a function is defined (input values)

Range – possible y values or output values

EXAMPLE 1



Domain is all the input value (those are on the horizontal axis). The furthest left is -3 and furthest right is 3. So $[-3, 3]$.

Range is y values or output values (those are on the vertical axis). The lowest value is -2 and the highest value is 1. So $[-2, 1]$.

EXAMPLE 2

Find the domain and range of $y = \sqrt{4 - x^2}$

Domain: Since the square root of a negative number is imaginary, the value of $4 - x^2$ must be positive so $4 - x^2 \geq 0$. Which means $-2 \leq x \leq 2$. So $[-2, 2]$

Range: The solution must be positive so $[0, \infty)$

Find the domain and range of each function. Write your answer in INTERVAL notation.

1. $f(x) = x^2 - 5$

2. $f(x) = -\sqrt{x + 3}$

3. $f(x) = 3 \sin x$

4. $f(x) = \frac{2}{x - 1}$

Complete the following table.

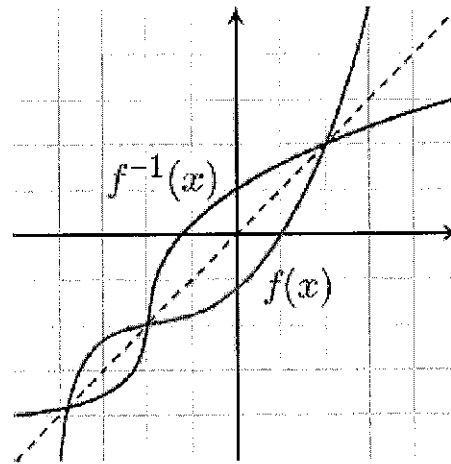
Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1, 7)$	

INVERSES:

To find the inverse of a function, simply switch the x and the y and solve for the new “ y ” value.
Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x) = \sqrt[3]{x+1}$	Rewrite $f(x)$ as y
$y = \sqrt[3]{x+1}$	Switch x and y
$x = \sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = (\sqrt[3]{y+1})^3$	Cube both sides
$x^3 = y+1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation



Find the inverse of each function.

1. $f(x) = 2x + 1$

2. $f(x) = \frac{x^2}{3}$

3. $f(x) = \frac{5}{x-2}$

4. $f(x) = \sqrt{4-x} + 1$

5. If $f(x)$ contains the point $(2, 7)$ then what is one point would be on $f^{-1}(x)$?

6. Write a complete sentence describing how $f(x)$ is related/compares to $f^{-1}(x)$.

DIFFERENT FORMS FO THE EQUATION OF A LINE:

Slope intercept form: $y = mx + b$

Vertical line: $x = c$ (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: $y = c$ (slope is 0)

* LEARN! We will use this formula frequently!

Example: Write a linear equation that has a slope of $\frac{1}{2}$ and passes through the point (2, -6)

Slope intercept form

$$y = \frac{1}{2}x + b$$

Plug in $\frac{1}{2}$ for m

$$-6 = \frac{1}{2}(2) + b$$

Plug in the given ordered

$$b = -7$$

Solve for b

$$y = \frac{1}{2}x - 7$$

Point-slope form

$$y + 6 = \frac{1}{2}(x - 2)$$

Plug in all variables

$$y = \frac{1}{2}x - 7$$

Solve for y

1. Write the equation of a line that passes through the point (5, -3) and has an undefined slope.
2. Write the equation of a line that passes through the point (-4, 2) and has zero slope.
3. Use point-slope form to write the equation of a line that passes through the point (2, 8) and is parallel to the equation $y = \frac{5}{6}x - 1$.
4. Use point-slope form to find a line perpendicular to $y = -2x + 9$ that passes through the point (4, 7).
5. Write the equation of the line that passes through the points (-3, 6) and (1, 2).
6. Write the equation of the line that has an x-intercept of (2, 0) and a y-intercept of (0, 3).

CONVERTING RADIANS \leftrightarrow DEGREES.

Use $\frac{180^\circ}{\pi \text{ radians}}$ to get rid of radians and
convert to degrees.

Use $\frac{\pi \text{ radians}}{180^\circ}$ to get rid of degrees and
convert to radians.

Convert to degrees.

1. $\frac{5\pi}{6}$

2. $\frac{4\pi}{5}$

3. 2.63 radians

Convert to radians.

4. 45°

5. -17°

6. 237°

Sketch each of the following angles in standard position.

(HINT: angles in standard position begin on the positive side of the x-axis and goes counterclockwise.)

7. $\frac{11\pi}{6}$

8. 230°

9. $\frac{-5\pi}{3}$

10. 1.8 radians

If $\tan x = \frac{5}{2}$ for $\pi \leq x \leq \frac{3\pi}{2}$, then find the following.

(HINT: draw a triangle in the correct quadrant, then use the Pythagorean theorem to find the missing side.)

11. $\sin x$

12. $\cos x$

13. $\cot x$

14. $\sec x$

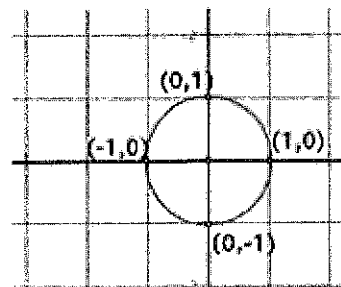
15. $\csc x$

UNIT CIRCLE:

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

Example: $\sin 90^\circ = 1$

$\cos \frac{\pi}{2} = 0$



Find the following.

1. $\sin 180^\circ$

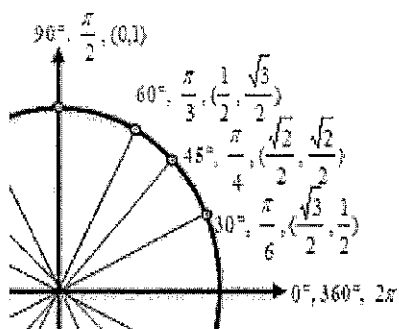
2. $\cos 270^\circ$

3. $\sin(-90^\circ)$

4. $\sin \pi$

5. $\cos \frac{3\pi}{2}$

6. $\cos(-\pi)$



You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as \sin/\cos or the slope of the line.

Examples:

$\sin \frac{\pi}{2} = 1$

$\cos \frac{\pi}{2} = 0$

$\tan \frac{\pi}{2} = \text{und}$

*You must have these memorized OR know how to calculate their values without the use of a calculator.

Find the EXACT value of each of the following.

7. $\sin \frac{\pi}{6}$

8. $\cos \frac{3\pi}{4}$

9. $\sin \left(\frac{-\pi}{2} \right)$

10. $\sin \left(\frac{5\pi}{4} \right)$

11. $\cos \frac{\pi}{4}$

12. $\cos(-\pi)$

13. $\cos \frac{\pi}{3}$

14. $\sin \left(\frac{5\pi}{6} \right)$

15. $\cos \frac{2\pi}{3}$

16. $\tan \frac{\pi}{4}$

17. $\tan \pi$

18. $\tan \frac{\pi}{3}$

19. $\cos \frac{4\pi}{3}$

20. $\sin \frac{11\pi}{6}$

21. $\tan \frac{7\pi}{4}$

22. $\sin \left(\frac{-\pi}{6} \right)$

TRIGONOMETRIC EQUATIONS:

Solve each equation for $0 \leq x < 2\pi$

1. $\sin x = \frac{-1}{2}$

2. $2 \cos x = \sqrt{3}$

3. $4 \sin^2 x = 3$

4. $2 \cos^2 x - \cos x - 1 = 0$

HINT: $\sin^2 x = (\sin x)^2$
If $x^2 = 25$, then $x = \pm 5$

HINT: factor

TRANSFORMATIONS OF FUNCTIONS:

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x - c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x + c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

- How does $g(x)$ differ from $f(x)$ if $f(x) = x^2$ and $g(x) = (x - 3)^2 + 1$?
- Write the equation for $g(x)$ if $f(x) = x^3$ and $g(x)$ has the shape of $f(x)$ but it is moved left 6 and reflected over the x-axis.
- If the ordered pair $(2, 4)$ is on $f(x)$, find one point on the following graphs:
 - $f(x) - 3$
 - $f(x - 3)$
 - $2f(x)$
 - $f(x - 2) + 1$
 - $-f(x)$
 - $\frac{1}{2} f(x)$

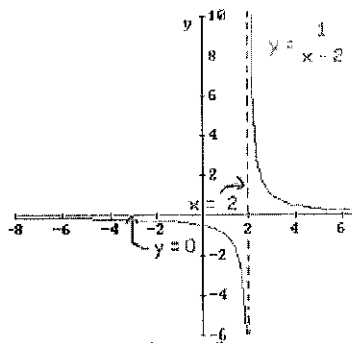
VERTICAL ASYPTOTES:

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).

Write a vertical asymptotes as a line in the form $x =$

Example: Find the vertical asymptote of $y = \frac{1}{x-2}$

Since when $x = 2$ the function is in the form $1/0$ then the vertical line $x = 2$ is a vertical asymptote of the function.



Find the vertical asymptotes for the following. (HINT: you may need to factor first!)

1. $f(x) = \frac{1}{x^2}$

2. $f(x) = \frac{x^2}{x^2 - 4}$

3. $f(x) = \frac{2 + x}{x^2(1 - x)}$

4. $f(x) = \frac{4 - x}{x^2 - 16}$

5. $f(x) = \frac{x - 1}{x^2 + x - 2}$

6. $f(x) = \frac{5x + 20}{x^2 - 16}$

HORIZONTAL ASYMPTOTES:

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Example: $y = \frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Example: $y = \frac{2x^2 + x - 1}{3x^2 + 4}$ (As x becomes very large or very negative the value of this function will approach $2/3$). Thus there is a horizontal asymptote at $y = \frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \frac{2x^3 + x - 1}{3x - 3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Determine all horizontal asymptotes.

1. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

2. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

3. $f(x) = \frac{4x^2}{3x^2 - 7}$

4. $f(x) = \frac{(2x - 5)^2}{x^2 - x}$

5. $f(x) = \frac{x^2 - x}{x + 1}$

EVEN AND ODD FUNCTIONS:

Recall:

Even functions are functions that are symmetric over the y-axis.

To determine algebraically we find out if $f(x) = f(-x)$

(*Think about it what happens to the coordinate $(x, f(x))$ when reflected across the y-axis*)

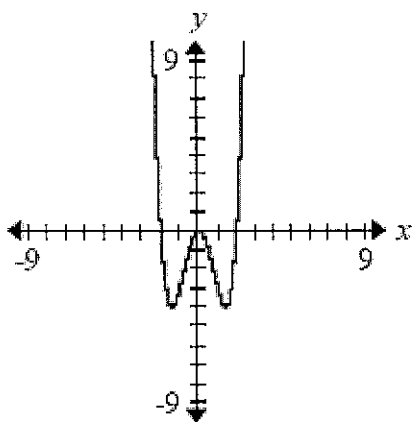
Odd functions are functions that are symmetric about the origin.

To determine algebraically we find out if $f(-x) = -f(x)$

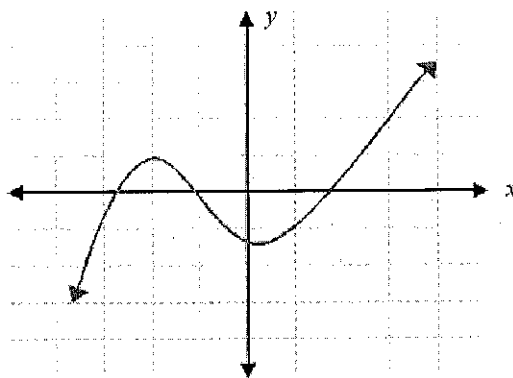
(*Think about it what happens to the coordinate $(x, f(x))$ when reflected over the origin*)

Determine if the following are even, odd or neither. For equations show work!!

1.



2.



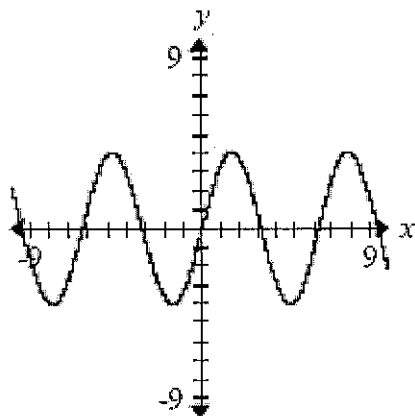
3. $f(x) = 2x^4 - 5x^2$

4. $f(x) = x^5 - 3x^3 + x$

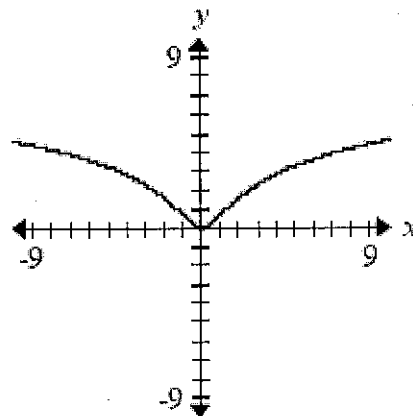
5. $g(x) = 2x^2 - 5x + 3$

6. $g(x) = 2\cos x$

7.



8.

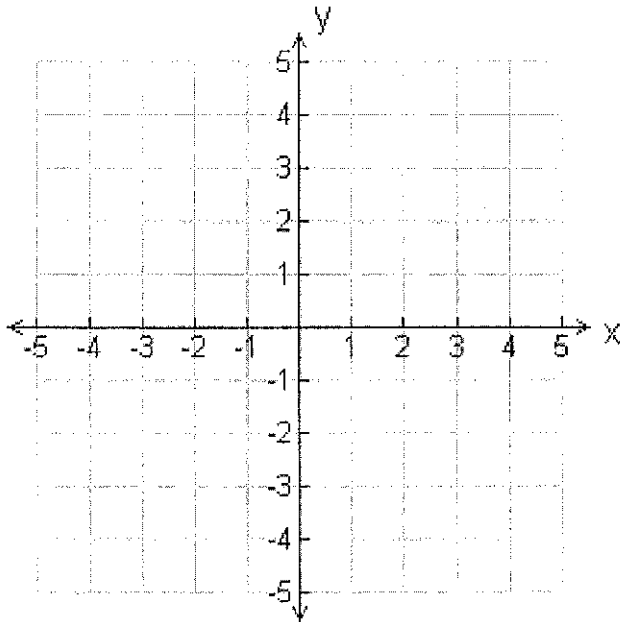


GRAPHS OF "OTHER" FUNCTIONS:

Graph the following.

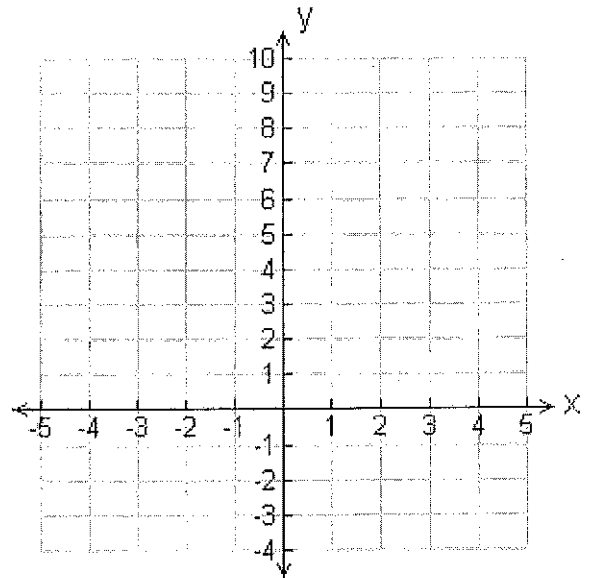
1.

$$f(x) = \begin{cases} 1 & x \leq 0 \\ -1 & x > 0 \end{cases}$$



2.

$$f(x) = \begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2) \\ -x+3 & (2, \infty) \end{cases}$$



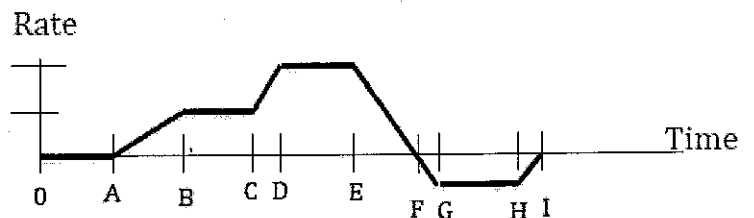
3. The rate at which water is filling and draining from a tank ($t > 0$) is represented by the graph below. A positive rate means that water is entering the tank, while a negative rate means the water is leaving the tank. State the intervals on which the following is true....

a) The volume of water is constant.

b) The volume of water is increasing.

c) The volume of water is decreasing.

d) The volume of water is increasing the fastest.



FACTORING

Factor completely.

1. $3x^4 + 4x^3 - x^2$

2. $x^2 - 7x + 12$

3. $2x^2 + 5x - 3$

4. $x^4 - 25$

5. $x^4 - 9x^2 + 8$

6. $x^3 + 6x^2 + 12x + 8$

7. $x^3 + 4x^2 - 2x - 8$

8. $5\cos^2 x - 5\sin^2 x + \cos x + \sin x$

Complete the following by factoring as indicated.

9. $2\sqrt{x} + 6x^{3/2} = 2\sqrt{x} (\quad)$

10. $\sin x + \tan x = \sin x (\quad)$

11. $\frac{1}{2x^2 + 4x} = \frac{1}{2x} (\quad)$

12. $\sqrt{x^2 + 1} - \frac{x^2}{\sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}} (\quad)$

13. $x^2 - 9 = (\quad) (\quad)$

14. $8x^3 + 14x^2 + 6x = 2x(\quad) (\quad)$

EXPONENTIAL FUNCTIONS

Example: Solve for x

$$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$$

$$(2^2)^{x+1} = (2^{-1})^{3x-2}$$

Get a common base

$$2^{2x+2} = 2^{-3x+2}$$

Simplify

$$2x + 2 = -3x + 2$$

Set exponents equal

$$x = 0$$

Solve for x

Solve for x:

1. $3^{3x+5} = 9^{2x+1}$

2. $\left(\frac{1}{9}\right)^x = 27^{2x+4}$

LOGARITHMS

The statement $y = b^x$ can be written as $x = \log_b y$
Remember a log is an exponent.

Example: Evaluate the following logarithms.

$$\log_2 8 = ?$$

In exponential form this is $2^? = 8$

Therefore $? = 3$

So $\log_2 8 = 3$

Evaluate the following:

1. $\log_7 7$

2. $\log_3 27$

3. $\log_3 \frac{1}{27}$

4. $\log_{25} 5$

5. $\log_9 1$

6. $\ln \sqrt{e}$

PROPERTIES OF LOGS

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

$$\log_b x^y = y \log_b x$$

$$b^{\log_b x} = x$$

Examples:

Expand $\log_4 16x$

$$\log_4 16 + \log_4 x$$

$$2 + \log_4 x$$

Condense $\ln y - 2 \ln R$

$$\ln y - \ln R^2$$

$$\ln \frac{y}{R^2}$$

Expand $\log_2 7x^5$

$$\log_2 7 + \log_2 x^5$$

$$\log_2 7 + 5 \log_2 x$$

Use the properties of logs to evaluate the following.

1. $\log_2 2^5$

2. $\ln e^3$

3. $\log_2 8^3$

4. $\log_3 \sqrt[3]{9}$

5. $2^{\log_2 10}$

6. $e^{\ln 8}$

7. $9 \ln e^2$

8. $\log_9 9^3$

9. $\log_{10} 25 + \log_{10} 4$

10. $\log_2 40 - \log_2 5$

11. $\log_2 (\sqrt{2})^5$

Solve for x.

12. $\log_2 16 = x$

13. $\log_3 1 = x$

14. $\log 10 = x$

14. $\ln 1 = x$

15. $\ln(e^3) = x$

16. $\ln x + \ln x = 0$

LIMITS

Option 1: evaluating (direct substitution)

example $\rightarrow \lim_{x \rightarrow -2} (x^2 + x - 6)$

Option 2: factoring out and reducing

example $\rightarrow \lim_{x \rightarrow -3} \frac{(x^2 + x - 6)}{x + 3}$

Option 3: rationalizing

example $\rightarrow \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$

Find the following limits.

1. $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$

2. $\lim_{x \rightarrow 1} \frac{4x + 5}{6x - 1}$

3. $\lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{x - 1}$

Write a sentence to describe the following.

4. How is a limit like two friends meeting at the mall?

5. What is meant by “right hand limits” and “left hand limits”?

6. For a limit to exist what has to be true about right hand and left hand limits?

7. Use the graph on the right to answer the questions below:

a) $\lim_{x \rightarrow 3^+} f(x)$

b) $\lim_{x \rightarrow 3^-} f(x)$

c) $\lim_{x \rightarrow 3} f(x)$

